An Adaptive Filtering Technique For Pilot Aided Transmission Systems

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ABSTRACT

Many recent papers have revisited the use of a pilot tone as a phase and amplitude reference for demodulation on fading channels. The receiver extracts the reference with a narrow-band pilot filter. Its bandwidth has an optimum value, representing a tradeoff between distortion of the Doppler-spread tone, at low values, and additive noise at high values. Since the optimum bandwidth varies with vehicle speed, some form of adaptivity is desirable. This paper is the first to address the issue of adjusting the filter bandwidth as speed changes. In addition, the adaptation technique is novel; the receiver selects the optimum member from a pre-calculated bank of stored filters, thereby reducing the search to one dimension. The technique can provide a 1.0 dB improvement for a normalized Doppler frequency of 2%; up to 2.0 dB can be gained when operating at a lower Doppler frequency. The algorithm is found to be very robust, in the sense that self-noise and decision feedback have little effect on its operation and the resulting bit error rate.

I. INTRODUCTION

Recent literature [1,2,3] has shown a renewed interest in pilot based calibration systems for combating the effect of multipath fading. Such systems transmit a pilot tone along with the data. The receiver then extracts the pilot with a narrowband pilot filter (PF) and the result is used as a phase and amplitude reference. One the main advantages of using such a technique is the elimination of the error floor. Some of the disadvantages are non-constant envelope of the resulting signal and the fact that part of the transmitter power is devoted to the pilot.

One of the main design tradeoffs of pilot based techniques is PF bandwidth. Ideally, the PF should be just wide enough to accommodate the fading spectrum. This tradeoff has been investigated recently [4]. If it is too narrow, the filter output cannot follow channel changes. The result is the reappearance of the error floor. If it is too wide, excessive noise will appear at the filter output which can lead to degradation in performance.

If the fading process were stationary, the PF could be optimized using Wiener filter theory. Unfortunately, the fading spectrum depends on the Doppler frequency (f_D), which is a function of the vehicle speed. Most literature of pilot tone techniques assumes the pilot filter to have bandwidth equal to the worst case, twice the maximum Doppler frequency. Using a worst case value means that at lower speeds, more noise is admitted into the filtered pilot than is necessary thus causing the BER to increase. This additional channel noise can be reduced if the PF bandwidth can follow or adapt to the Doppler frequency. This problem of adapting the PF to the Doppler frequency has not been addressed previously.

There are many adaptive filtering schemes in use today. Most popular are the least mean square (LMS) and recursive least square (RLS) algorithms using either the transversal or lattice structures. These algorithms need to update each filter coefficient and thereby perform a multidimensional adaptation. In the pilot filter application, the optimum filter for any Doppler frequency is the Wiener filter. Our approach to the problem is to store a pre-calculated bank of filters and simply select one as PF. In this way the adaptation process has been reduced to a single dimension. We call the technique the filter switching algorithm (FSA). This paper gives a description, an analytical model, and the resulting performance.

II. SYSTEM MODEL

The system model of a pilot based calibration system using the filter switching algorithm is shown in Figure 1. All signals described in this paper are assumed to be in complex envelope representations.

For simplicity, we assume that the transmitted signal is BPSK and Manchester coded to create the spectral null for the pilot. The transmitted signal is split between the data signal, s(t) and the pilot having amplitude a. The transmitted complex envelope is given by:

$$z(t) = s(t) + a \tag{1}$$

where the data signal is defined by:

$$s(t) = A \sum_{i=-\infty}^{\infty} b_i \, p(t-iT) \tag{2}$$

p(t) is assumed to be an unit energy pulse such that $\int |p(t)|^2 \, dt = 1$. b_i is the binary data which can assume the values: +1 or -1. If we denote the bit rate by R_b and the ratio of pilot tone power to data signal power by r, then r is given by:

$$r = \frac{a^2}{A^2 R_b} \tag{3}$$

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The transmitted complex envelope z(t) is multiplied by a time-varying complex Gaussian gain g(t) representing the effect of multipath fading. Complex white Gaussian noise n(t) is added to form the received signal r(t). The present paper assumes no frequency offset between the transmit and receive oscillators, although it has been considered elsewhere [5].

The spectrum of g(t), denoted by S_g, can be written as:

$$S_g(f) = \sigma_g^2 \, \tilde{S}_g(f) \qquad (4)$$

where σ_g^2 is the total power gain and $\tilde{S}_g(f)$ is the fade spectrum normalized to unit power.

The scattering due to multipath reflections is assumed to be isotropic so that the normalized fade spectrum is given by:

$$\tilde{S}_g(f) = \frac{1}{\pi \sqrt{f_D^2 - f^2}} \qquad (5)$$

The received signal is split into two branches, one for the processing of the data signal and the other for the processing of the pilot. At the data branch, the received signal r(t) passes through a unit energy filter which is matched to the modulating pulse shape. Delay is added to the matched filter output in order to compensate for the extra delay in the pilot branch. Assuming that the Doppler frequency is much less than the bit rate, the delayed filter output is given by:

$$u(kT) = g(kT) \, A \, b_k + n(kT) \qquad (6)$$

where n(kT) is additive white Gaussian noise (AWGN) with variance N_0 and T is the sampling period $(T = 1/R_b)$.

To obtain a performance reference for the FSA, we remove the data dependence in the delayed matched filter output u(kT) by multiplying it with the demodulated data, $\hat{b}_k$. At low BER, $\hat{b}_k \approx b_k$ so that $\hat{u}(kT)$ can be approximated as:

$$\hat{u}(kT) \approx A \, g(kT) + n_u(kT) \qquad (7)$$

where $n_u(kT)$ is Gaussian with variance N_0. The error introduced by this decision direction has been found to be small [5].

The pilot processing branch consists of an integrate and dump filter, a bank of filters and an adaptation loop. The integrate and dump (I & D) filter removes the data modulation present in the received signal. The output of the I & D filter is then sampled at the bit rate before entering the pilot filter. The pilot filters have a low-pass characteristic for complex envelope representation. For ease of implementation, they are structured as a cascade of a fixed moving window averager (MWA) and a reduced coefficient filter. The MWA has a rectangular impulse response spanning several symbols, and its output is sampled at the symbol rate. The reduced coefficient filters have non-zero coefficients spaced by the length of the MWA. The combined effect is to approximate the desired impulse response with a staircase. Each reduced coefficient filter has a corresponding gradient filter in the adaptation loop.

Denote the time response of the combined MWA and reduced coefficient filter by $h_p(i, kT)$ where i is the index of the particular filter in the filter bank. The filter output is:

$$w(kT) = a \, [g(kT) * h_p(i, kT)] + n_w(kT) \qquad (8)$$

where $*$ denotes convolution and $n_w(kT)$ is Gaussian with variance KN_0. w(kT) is conjugated and multiplied by the delayed matched filter output u(t) to form the phase corrected data decision variable d(kT), given by:

$$d(kT) = Re[u(kT) \, w(kT)^*] \qquad (9)$$

with $[\cdot]^*$ representing conjugation.

Clearly, the BER improves with increasing correlation between u(kT) and w(kT); in fact, for Rayleigh fading channels, it can be shown that [4]:

$$P_e = \frac{1}{2} \left[1 - \sqrt{\rho_r^2/(1-\rho_i^2)} \right] \qquad (10)$$

where ρ_r and ρ_i are the real and imaginary parts of the complex correlation coefficient ρ. ρ is in turn given by [4]:

$$\rho = \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{S}_g(e^{j(\omega-\omega_0)}) \, H_p(e^{j\omega})^* \, d\omega \right\} \cdot \left\{ 1 + (1+r)\frac{N_0}{E_b} \right\}^{-1/2}$$

$$\cdot \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{S}_g(e^{j(\omega-\omega_0)}) \, |H_p(e^{j\omega})|^2 \, d\omega + (1+r)\frac{N_0}{E_b}\frac{B_p}{rR_b} \right\}^{-1/2} \qquad (11)$$

where $H_p(e^{j\omega})$ denotes the frequency response of the pilot filter having noise bandwidth B_p, and E_b represents the total received energy per bit. For a pilot filter with a real frequency response, $\rho_i = 0$, and the expression for BER reduces to:

$$P_e = \frac{(1-\rho)}{2} \qquad (12)$$

The adaptation loop provides the mechanism for selecting the filter that maximizes ρ. The loop consists of a bank of gradient filters and a sample averager. The gradient filter has a unit impulse response which equals the difference between impulse responses of two reduced coefficient filters. We denote the impulse response of the combined MWA and gradient filter as $\Delta h_p(i,kT)$ given by:

$$\Delta h_p(i,kT) = h_p(i+1,kT) - h_p(i,kT) \qquad (13)$$

The gradient filter output is:

$$v(kT) = a \, [g(kT) * \Delta h_p(i,kT)] + n_v(kT) \qquad (14)$$

v(t) is conjugated and multiplied by the decision corrected data signal $\hat{u}(kT)$ to form an instantaneous cross-correlation y(kT). y(kT) is averaged by a sample averager with length N to give a sampled cross-correlation q(kT). The control variable, x(kT), which determines the next filter to be used, is given by:

$$x(kT) = Re[q(kT)]$$
$$= \frac{1}{N} \sum_{m=0}^{N-1} Re[y(mT)]$$

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$$= \frac{1}{N} \sum_{m=0}^{N-1} \text{Re}[\hat{u}(mT)\, v(mT)^*] \qquad (15)$$

Samples of $q(kT)$ are taken in contiguous blocks (of size N) such that no overlapping samples are used in forming $x(kT)$. The next section describes the filter operation.

III. FILTER SWITCHING ALGORITHM

A. Filter Operation

The bank of pilot filters consists of filters of different band-widths from which one is selected for pilot recovery. The filter set is arranged in order of increasing bandwidth. During initial operation, the filter with the widest bandwidth (equal to f_{Dmax}) is selected. The control variable $x(k)$ is then evaluated at some fixed time interval which determines whether to switch to a wider or narrower bandwidth filter. Assuming that the filters are ideal rectangular, then after convergence the bandwidth jitters about the optimal value of f_D. Although it is possible to switch from the filter in use to any other filter in the set, we allow only a single step at a time, up or down in order to simplify analysis.

Some of the parameters which are important to the performance of the algorithm are: filter shape, filter length and bandwidth spacing. Selection of filter length involves the usual tradeoff between performance, computation complexity and delay. Smaller bandwidth spacing provides some improvement. However, the number of filters, and hence the amount of storage space required, is proportional to the bandwidth spacing. Again a tradeoff is needed.

B. Derivation of the Filter Switching Algorithm

From (12), the adaptation should maximize ρ. However, (11) shows this to be computationally demanding. Instead, we maximize the simpler cross-correlation C, defined as:

$$C = \sigma_{\hat{u}w}^2 = E[\hat{u}(k)\, w(k)^*] \qquad (16)$$

In [5], it is shown that C and ρ have maxima at the same point, so there is no loss in using C.

The filter switching algorithm is an application of the steepest descent algorithm [6]. At each update, it takes a step in a direction opposite to the gradient of C:

$$B_p(n+1) = B_p(n) - \mu\, \frac{\partial C}{\partial B_p} \qquad (17)$$

Approximating $\frac{\partial C}{\partial B_p}$ by $\frac{\Delta C}{\Delta B_p}$ and letting $\mu' = \frac{\mu}{\Delta B_p}$, then:

$$B_p(n+1) = B_p(n) - \mu'\, \Delta C \qquad (18)$$

where ΔB_p represents a step increment in B_p and ΔC represents the resulting change in C. Since the cross-correlation is linear with respect to the filter impulse response, ΔC is given by:

$$\Delta C = E[\hat{u}(k)\, v(k)^*] \qquad (19)$$

We now quantize B_p so that only a finite number of filters is required. Next we remove the dependence of $v(k)$ on the data by multiplying $u(k)$ by the demodulated data $\hat{b}_k$. To simplify the algorithm further, ΔC is replaced by $\text{sign}(\Delta C)$; i.e. we will allow only transitions one step up or down, as noted earlier. For practical purposes, we approximate ΔC by a sample mean, and use only its real part so that the control variable becomes $x(k)$, as defined by (15). The resulting algorithm is then to calculate $x(k)$:

$$\text{If } x(k) > 0, \text{ then } B_p(k+1) = B_p(k) + \Delta B_p$$
$$\text{else } B_p(k+1) = B_p(k) - \Delta B_p. \qquad (20)$$

The mean and the variance of $x(k)$ can be computed exactly from the filter shape and the fading spectrum [5]. However, the probability density function of $x(k)$ appears to be intractable. We found it convenient in calculating the Markov transition probabilities below to approximate it as Gaussian. This approximation is loosely based on the central limit theorem because samples in $q(k)$ separated by more a few fade cycles are essentially uncorrelated, and $x(k)$ is a sum of several such samples. Simulations showed generally good agreement.

IV. FILTER SWITCHING ANALYSIS

This section provides an analytical model for the filter switching algorithm from which the convergence speed and the average BER can be derived.

A. Markov Chain Model

Since the selection of filters depends on $x(k)$, the index associated with each filter in the filter banks is a discrete time random variable. With the assumption that each $x(k)$ value used in the switching decision is independent of the previous values, the filter index i is described by a discrete time Markov chain. Given the transition probabilities, a set of steady state probabilities can be computed if the Markov chain is irreducible. Since it is possible to compute the BER for a particular f_D and pilot filter used, then the average BER is simply:

$$E[P_e] = \sum_{i=1}^{M} P_e(i)\, v_i \qquad (21)$$

where $P_e(i)$ denotes the BER for filter i computed using (11) and (12), v_i denotes the steady state probability for filter state i, and M is the number of filters in the ensemble.

An illustration of the Markov chain model described in the previous section is given in Figure 2, with transition probabilities represented by b_i and d_i. Unfortunately, the model as shown in Figure 3 is not irreducible, since odd and even states alternate. However, the original chain can be split into two, an even and an odd states chain. If we group every two transitions on the original chain into one transition on one of the even or odd state chains and consider even and odd step transitions separately, then this dual Markov chain model (shown in Figure 3 for M even) is identical to that shown in Figure 2.

Assuming that the transition probability matrix of the original chain is $\mathbf{P}$, the new chains can be shown to be irreducible with transition probability matrix $\mathbf{P}^2$. Their steady state probabilities exist and they can be easily computed.

The transition probability b_i is derived from the pdf of $x(k)$ as follows. The adaptation algorithm dictates that if the process is at state i, it will switch to the next higher state if $x(k) > 0$; otherwise it will switch to the next lower state. Thus the probability of switch from state i to state i+1 is simply given by:

$$b_i = p_{i,i+1} = Pr(x(k) > 0) \qquad (22)$$

Since we approximate $x(k)$ in (15) as Gaussian, this probability is easily obtained from the mean and variance of $x(k)$.

Convergence time in this adaptation method is the time taken to reach the optimum filter after being released from a specific starting state. More precisely, it is the expected first passage time; a detailed derivation of the first passage time given the transition matrix can be found in [7].

B. Step margin

Because we restrict transitions to one step up or down (no self loops), the filter bandwidth will dither about the optimum value, even when converged. Unfortunately, if the filter bandwidth is too small, the BER will rise sharply. This indicates that some margin of safety should be added to the pilot filter bandwidth chosen. The idea has been incorporated in the filter switching algorithm by selecting a pilot filter with bandwidth which is a number of steps wider than that of the filter corresponding to the gradient filter in use. We call this margin of safety (in number of steps) the step margin.

C. Simulation Results

Simulations were performed to determine the accuracy of the model and also to investigate the effect of self noise. Figure 4 shows the steady state probabilities from the simulated and computed values for f_D of 2.08 % (50 Hz at 2400 bps) and a bandwidth increment of 10 Hz. The simulation also included the effects of self-noise and decision errors. There is good agreement. We conclude that the Gaussian approximation of $x(k)$, and the neglect of self-noise and decision errors in the analytical model, are justified.

V. BER & CONVERGENCE PERFORMANCE COMPUTED BASED ON THE MARKOV MODEL

Figure 5 shows the BER as a function of E_b/N_o for different Doppler frequencies, assuming BPSK signalling. A cross-correlation sample size N of 300 was used in each case. The BER for a lower f_D is smaller than for a higher f_D because of a smaller average bandwidth. As a reference, Figure 5 shows the BER curve for a non-adaptive pilot tone calibration system using a rectangular PF with bandwidth of 150 Hz. At a BER of 10^{-2}, improvement over the non-adaptive system is about 0.3 dB for 100 Hz Doppler, 1.0 dB for 50 Hz Doppler and almost 2.0 dB for a 10 Hz Doppler.

Convergence time in the case of 100 Hz Doppler was 21 steps from the last state (largest B_p) and 32 steps from the first state (smallest B_p). Convergence from the first state corresponds to a vehicle accelerating from standstill to 127 kph (assuming an 850 MHz carrier). Recall that one step corresponds to the 300 bits of the sample averager, or 125 ms at 2400 bps. The 32 steps of the convergence time therefore corresponds to 4 seconds, and is certainly adequate to track the acceleration. In

fact, in terms of f_D per second, the convergence speed is 25 Hz/s for a 2400 bps system. Realistically, the fastest change in f_D which can be expected is only about 15.5 Hz/s (corresponding to at an acceleration of 0-100 kph in 5 sec.).

Note that a cross-correlation sample size of 300 was used in the results presented. It has been found that a sample size of 150 can be used to achieve similar BER performance and hence double the convergence speed. In addition, at higher data rates, convergence speed is correspondingly faster.

VI. CONCLUSIONS

The paper has demonstrated that adaptivity in pilot filter bandwidth can produce a significant improvement in the BER of a pilot aided calibration system. The adaptation technique is also novel, and it can be applied to more general situations. Simulation results were presented which showed good agreement with analytical results. It was shown that up to 2.0 dB can be gained using this technique, as compared to a non-adaptive receiver.

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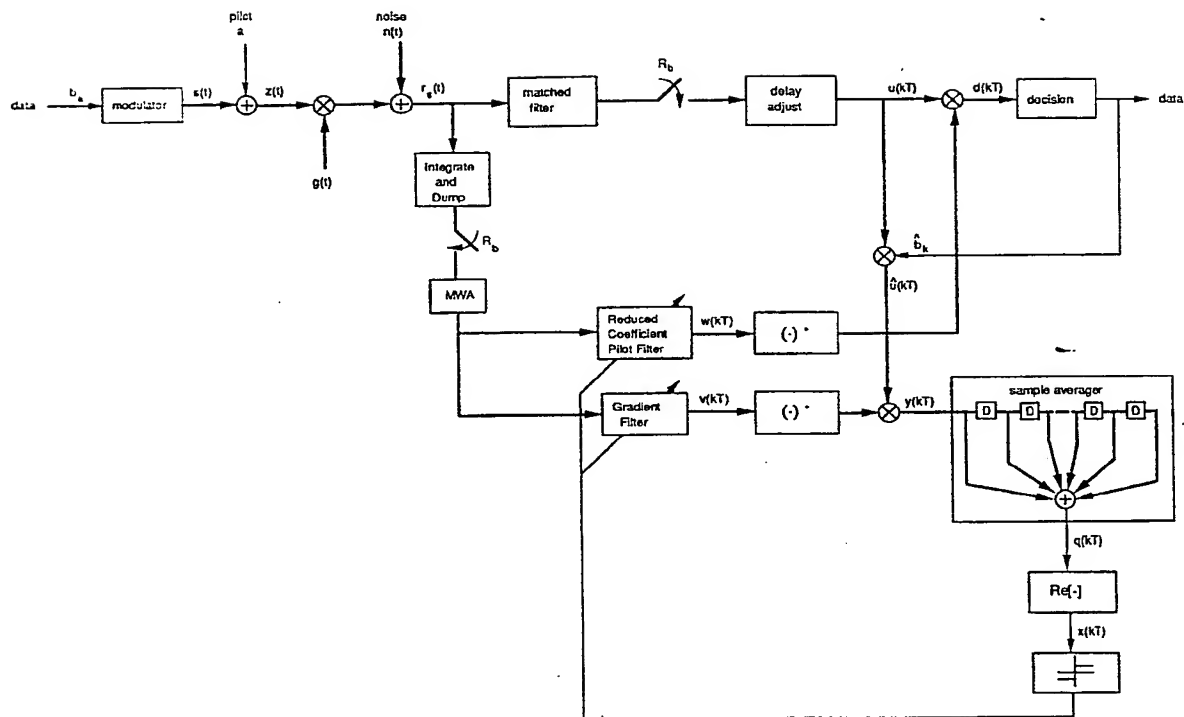[7] J. G. Kemeny and J. L. Snell, Finite Markov Chains, D. Van Nostrand Company Ltd., 1960.
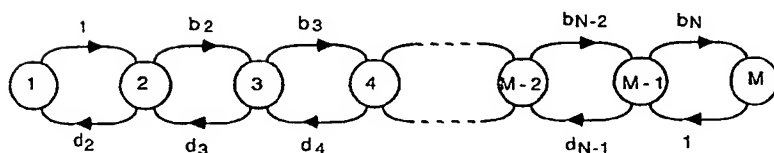
Figure 1 - System Model

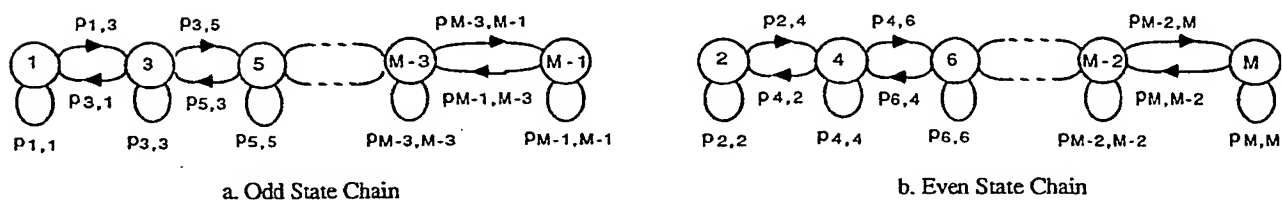Figure 2 - Markov Chain Model of the Filter Switching Process

a. Odd State Chain

b. Even State Chain

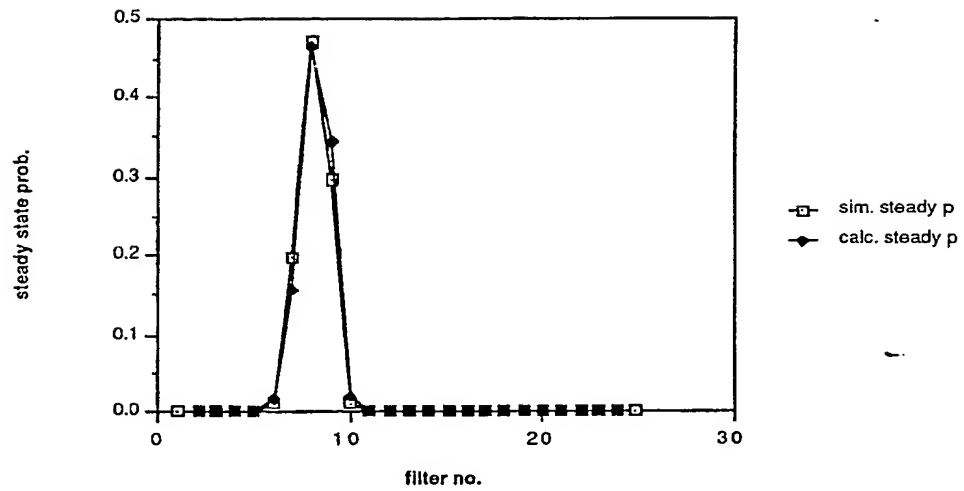Figure 3 - Dual Markov Chain Model of the Filter Switching Process for M even

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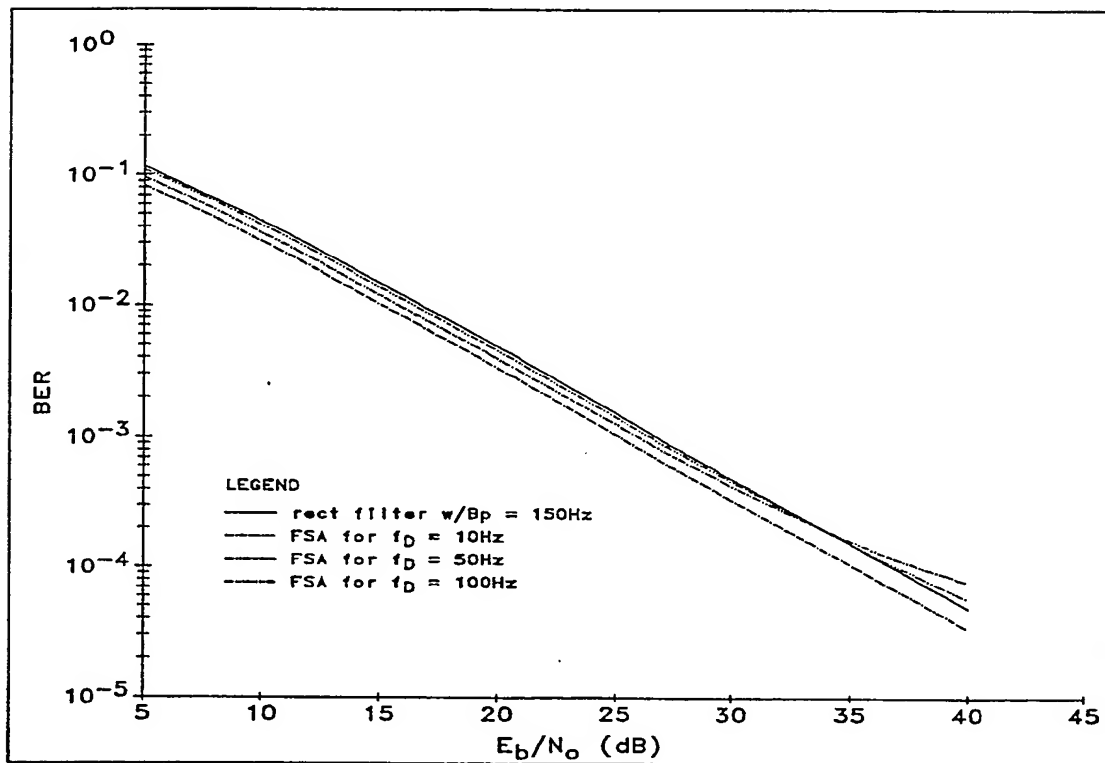Figure 4 - Calculated vs. Simulated Steady State Probabilities

Figure 5 - Average BER Performance of the FSA at Various Doppler Frequencies

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